International Journal of Theoretical Physics, Vol. 22 No. 11, 1983

Quantum Conformal Fluctuations and Stationary States

T. Padmanabhan

Astrophysics Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400 005, India

Received April 7, 1982

Conformal fluctuations serve as a powerful tool to study the nature of quantum gravity. They lead, in a natural fashion, to the concept of stationary states for the quantum geometry. We attempt to incorporate the effect of conformal fluctuations into the background metric and matter. A modified set of equations, including the effect of conformal fluctuations, is presented and the solutions are discussed. It is shown that matter-free vacuum is unstable to conformal fluctuations. A scenario for creation of matter is indicated.

1. INTRODUCTION

At the turn of this century, physics was facing a crisis: the existing formalism of classical physics predicted that the electron in the hydrogen atom will spiral down to the nucleus. This collapse was averted with the advent of quantum physics. Quantum theory introduced the concept of definite, stable stationary states for the electron to exist in, thereby rendering meaningless the unique classical trajectory that spirals down to the proton.

It was realized in the sixties (Hawking and Ellis, 1973) that classical physics predicts a much more serious collapse also—the collapse of space-time geometry at a singularity. Once again, it is necessary to see whether a quantum version of the theory would avoid the classical singularity. (For a discussion of this analogy, see Misner, Thorne, and Wheeler, 1973.)

Lack of a concrete theory of quantum gravity has prevented physicists from answering the question unequivocally. Quantum gravity shakes the foundations of space-time structure—and hence, the concept of causal separation—and leads to complicated conceptual problems.

0020-7748/83/1100-1023$03.00/0

Recently, an attempt was made to quantize gravity, in which the above problem could be attacked effectively (see, for example, Narlikar, 1981; Padmanabhan and Narlikar, 1981). In this approach, the path integral formalism is used to quantize the conformal part of the geometry—which is one of the two degrees of freedom of gravity—freezing the other degree of freedom at the classical value. In cases like the Robertson–Walker universe, which possess a high degree of symmetry, there is only one degree of freedom (Ryan, 1972) and this approach can be used with confidence.

Such an analysis has shown that quantum transitions to nonsingular space-times are overwhelmingly probable near a classical singularity (Padmanabhan and Narlikar, 1982a). It has also been shown that conformal quantum fluctuations avoid the singularity and set a lower bound to the length scale (Padmanabhan and Narlikar, 1981). Thus the situation parallels the case of the hydrogen atom to a remarkable extent, as far as this approach is concerned.

The question arises as to how far this analogy can be pursued. What is the nature of the stationary states of geometry? What does the simplest quantum gravity structure look like? Can one develop a full-fledged theory of quantum gravity along these lines? We discuss these aspects in the present paper.

In the first few sections of the paper, we discuss the nature of stationary states corresponding to simple space-time geometries. This analysis confirms the previous conclusions. Later, we present an attempt at modifying the classical equations, taking into account the quantum corrections (at least at a semiclassical level). The solutions of this set of equations have some novel features, which we discuss in the last part of the paper.

The logistics and details of this approach of quantum conformal fluctuations are discussed in the papers referred to previously and will not be repeated here. We shall indicate the basic formalism and borrow the necessary results from previous works.

2. QUANTUM STATIONARY STATES

2.1. Basic Formalism. Consider a four-dimensional space-time, which is foliated by a set of three-dimensional spacelike hypersurfaces $\Sigma(t)$, parametrized by a global timelike coordinate t. Classical evolution of the space-time geometry is determined by the Einstein equations, which can be obtained from the variation of the action (units: $c=1$)

$$J_{\text{tot}} = \frac{1}{16\pi G}\int R(-g)^{1/2} d^4x + J_m + J_{\text{Hawking}} \qquad (1)$$

The space-time geometry must be varied in the four-dimensional region, sandwiched between two spacelike hypersurfaces Σ_1 and Σ_2, on which the 3-geometries, $3\mathcal{G}$, are fixed. In equation (1), J_m stands for the action for matter; J_{Hawking} is a surface counterterm required to remove the second derivatives in the Einstein action [for details, see, e.g., Hawking (1979).] Once the 3-geometries are specified on Σ_1 and Σ_2, Einstein's equation uniquely specifies the space-time geometry.

This uniqueness, however, is lost in the quantum version. Quantum gravity will provide the probability amplitude for transition from one $3\mathcal{G}_1$ at Σ_1 to another 3-geometry $3\mathcal{G}_2$ at Σ_2, in the form of a path integral,

$$K[3\mathcal{G}_2\Sigma_2;\, 3\mathcal{G}_1\Sigma_1] = \int \mathcal{D}\mathcal{G} \exp\left(\frac{i}{\hbar}[J_{\text{tot}}]\right) \tag{2}$$

Expressing J_{tot} in terms of true degrees of freedom and choosing a suitable integration measure for these variables can be, in general, a formidable problem. However, one can rigorously define the kernel when the attention is confined to the conformal part of the geometry as a quantum variable [which becomes a rigorous theory when the space-time has a high level of symmetry, i.e., when the superspace is one dimensional, see Padmanabhan (1981)]. Following the earlier works, we shall write the metric $g_{ik}(\mathbf{x}, t)$, separating the conformal part, as

$$g_{ik}(\mathbf{x}, t) = \Omega^2(x)\bar{g}_{ik}(x) \tag{3}$$

and treat $\Omega(\mathbf{x}, t)$ as a quantum variable, with a given background $\bar{g}_{ik}$. The kernel can now be written as

$$K[\Omega_2(\mathbf{x}), t_2;\, \Omega_1(\mathbf{x}), t_1] = \int \mathcal{D}\Omega(x) \exp\left(\frac{i}{\hbar}J\right) \tag{4}$$

where the path integral is over the c^2 functions, $\Omega(\mathbf{x}, t)$, satisfying the boundary conditions; and J is given by (prime indicates quantities corresponding to $\bar{g}_{ik}$)

$$J[\Omega] = \frac{1}{16\pi G}\int \left(\bar{R}\Omega^2 - 6\Omega_i\Omega^i\right)(-g)^{1/2}\, d^4x + J_m \tag{5}$$

[see, for a derivation, Narlikar (1981)]. Notice that the action J is quadratic in $\Omega(x)$, because of which the functional integral is mathematically well defined. Such a situation allows one to write down a unique "Schrödinger equation" for the quantum variable Ω, and discuss the stationary states of the system (see Feynman and Hibbs, 1965). In the classical limit, the expectation value for Ω^2 must be used to interpret equation (3). Essentially, we have to study the quantum theory of a scalar field $\Omega(x)$ described by the action in equation (5). We shall discuss this situation step by step in the coming sections, starting from the simplest.

2.2. Vacuum Fluctuations of Gravity. It has been suggested many times that small-scale space-time behavior is predominated by the fluctuations of quantum gravity. The present formalism can be used to demonstrate this aspect explicitly. Consider a space-time with a flat background metric [i.e., $\bar{g}_{ik} = \text{dia}(1, -1, -1, -1)$]. The conformal fluctuations are governed by the action

$$J = -\frac{3}{8\pi G}\int \left(\Omega_i \Omega^i\right) d^4x \tag{6}$$

By the standard technique of Fourier transform, we write

$$\Omega(\mathbf{x}, t) = \int a_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}} \frac{d^3\mathbf{k}}{(2\pi)^3} \tag{7}$$

reducing the action in equation (6) to that of a set of harmonic oscillators (see Feynman and Hibbs, 1965), with an extra negative sign,

$$J = -\frac{3}{8\pi G}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\int_{t_1}^{t_2} dt\left[|\dot{a}_{\mathbf{k}}^2| - |\mathbf{k}|^2 |a_{\mathbf{k}}^2|\right] \tag{8}$$

Since the independent harmonic oscillators can be trivially quantized (in spite of the overall sign change, which is arbitrary as long as there is no interaction) the problem can be completely solved. The ground state for such a system (corresponding to the ground state of all the oscillators), can be written in many different ways to give the probability for finding different quantum conformal factors in the classical flat space. One physically transparent form [compare with the corresponding expression for electromagnetic field, discussed by Wheeler in, for example, Wheeler (1964)] for the ground state functional of the gravitational field is

$$\psi[\Omega] = N\exp\left[-\frac{3}{8\pi^3}\frac{1}{L_p^2}\iint d^3\mathbf{x}\, d^3\mathbf{y}\frac{(\nabla_{\mathbf{x}}\Omega)\cdot(\nabla_{\mathbf{y}}\Omega)}{|\mathbf{x}-\mathbf{y}|^2}\right] \tag{9}$$

Here $L_p = (G\hbar/c^3)^{1/2}$ is the Planck length and we have expressed the functional ψ in terms of the physically relevant gradients of Ω (similar to "electromagnetic field"; Ω corresponds to the "potential"). It is clear that quantum gravity affects the space-time structure at distances of the order of the Planck length. One can evaluate the expectation value of Ω^2 in the ground state and use equation (3) to discuss the effects of quantum gravity.

The harmonic oscillators corresponding to the $\Omega(\mathbf{x}, t)$ can exist in various stationary states (Fock basis for "gravitation") other than ground state. Each such configuration corresponds to a stationary state for quantum geometry, representing different levels of "excitation." This is the simplest form of stationary states we come across.

2.3. Fluctuations in a Space-Time of Maximal Symmetry. We consider the conformal fluctuations in a space-time which has the maximal symmetry—isotropy and homogeneity. Such a symmetry restricts the conformal part to depend only on time (see Narlikar, 1978), so that the metric has the form

$$ds^2 = \langle\Omega^2(t)\rangle\left[dt^2 - \frac{dr^2}{(1 - r^2/a^2)} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \qquad (10)$$

Here a is a constant, real or imaginary, corresponding to the "closed" and "open" space-times. It turns out that this fact does not affect the ensuing discussion critically. Hence we shall discuss the "closed" model (real a) and merely present the results for "open" model. Classically, the metric in equation (10) represents a Robertson–Walker space-time, with a suitable source. We are now interested in a situation where $\Omega(t)$ is a quantum variable [the classical limit in equation (10) must be obtained by a suitable expectation value of Ω^2, as indicated], and more specifically in the stationary states of this geometry.

This can be obtained using the formalism previously developed. The action now has the form

$$J = \frac{3\mathcal{V}}{8\pi G}\int_{t_1}^{t_2} dt\left(\frac{\Omega^2}{a^2} - \dot{\Omega}^2\right) + J_m \qquad (11)$$

where $\mathcal{V}$ is the proper volume of the region where the fluctuations are considered. (For the closed model, this may be taken to be the total volume.) As for the source, we shall take an isotropic radiation field, in which case (because of conformal invariance) J_m will be independent of Ω. (Slight modification is required if the source consists of dust; see below.) Treating Ω as a quantum variable, one can write down the Hamiltonian for the system as

$$H = -\left(-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial q^2} + \frac{1}{2}M\omega^2 q^2\right) \tag{12}$$

where (normal units)

$$q = a\Omega; \qquad \omega = \frac{c}{a}; \qquad M = \frac{\mathcal{V}c^2}{Ga^2} \tag{13}$$

The Hamiltonian corresponds to that of a harmonic oscillator. (The overall minus sign merely changes the signs of "energy" eigenvalues. In ordinary quantum mechanics the physically relevant energy is positive. In our present case the "energy" is merely a mathematical parameter used in separating the variables, and hence, can take any sign.) The stationary states are labeled by an integer n and are given by the usual Hermite polynomials. The expectation value for Ω in the nth quantum state is given by

$$\langle\Omega^2\rangle_n = \frac{\hbar}{M\omega a^2}\left(n + \frac{1}{2}\right) = \left(\frac{L_p}{a}\right)^2 \frac{1}{f}\left(n + \frac{1}{2}\right) \tag{14}$$

where $f = \mathcal{V}/a^3$ and L_p is the Planck length. After a change of scale, the space-time in a stationary state is represented by

$$ds^2 = f^{-1}L_p^2(n + \tfrac{1}{2})\left[d\eta^2 - d\chi^2 - \sin^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \tag{15}$$

Thus the space-time length scale is quantized in the units of Planck length with a nonzero lower bound.

It is necessary to examine the classical limit of this quantized system. The classical equations of motion derived from the action (11) have the solution

$$\Omega^{\mathrm{cl}} = \Omega_0 \sin \omega t \tag{16}$$

(with a suitable choice for origin of time) which correctly represents a radiation-filled Robertson–Walker model (see, e.g., Weinberg, 1972). The correspondence with classical theory is most pronounced in a quantum state where the expectation value of Ω has the classical value, i.e.,

$$\langle\Omega\rangle = \Omega_0 \sin \omega t = \Omega^{\mathrm{cl}} \tag{17}$$

It is well known that for the Harmonic oscillator potential there exists a nondispersive coherent state with classical expectation value (see, for details, Schiff, 1968; Padmanabhan, 1981). This state has the probability function given by

$$|\psi|^2 = c\exp\left[-\frac{M\omega a^2}{2\hbar}(\Omega - \Omega_0 \sin\omega t)^2\right] \tag{18}$$

which is a Gaussian peaked at the classical solution in equation (16) (with a suitable choice for the zero of time). This is the closest approximation classical limit. If one assumes that the universe is in this quantum state, the expectation value of Ω^2 can be calculated from equation (18), leading to the metric

$$ds^2 = \left(a^2\sin^2\eta + f^{-1}L_p^2\right)\left[d\eta^2 - d\chi^2 - \sin\chi\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right] \tag{19}$$

The classical evolution is modified by quantum corrections (the term $f^{-1}L_p^2$) near the singularity. Because of the lower bound in the length scale the evolution cannot proceed to dimensions below the Planck length.

A very similar analysis can be performed when the source consists of dust rather than radiation. The evolution of the geometry is described by

$$ds^2 = \left[a^2(1-\cos\eta)^2 + f^{-1}L_p^2\right]\left[d\eta^2 - d\chi^2 - \sin^2\chi\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right] \tag{20}$$

[see Padmanabhan (1982) for details]. The major conclusions remain unchanged in the case of the open model also.

One might wonder as to what is the correspondence between the stationary states of equation (15) and coherent state in equation (19). Such a problem exists even for, say, a simple pendulum: a bob suspended by a string will have a sinusoidal oscillation of position in time [just as the radiation-filled universe has a classical evolution equation (16)] while the quantum theory predicts a set of stationary states. The correspondence between the two involves questions of measurement. Quantum mechanically the "state of a system" has a meaning only after a measurement has been performed on the system. It is the act of measurement which prepares the system in that state. It is not clear how to describe a "quantum measurement of the universe," when the observer is part of the system. Further progress must await a clarification of this issue.

3. ATTEMPT FOR A MORE COMPLETE THEORY

3.1. Introduction. Gravity has two independent degrees of freedom of which the conformal part is directly related to one (see Misner, Thorne, and Wheeler, 1973; Isenberg and Wheeler, 1979). Robertson–Walker universes, however, are completely described by the conformal part and our formalism can be used with confidence. In a general case, our approach brings out only *some* features of quantum gravity. It is necessary to see whether the formalism can be extended further, at least as a valid approximation.

More specifically, one would like to answer the following questions: (i) How is the background metric $\bar{g}_{ik}$ determined? So far we have merely assumed a particular form for the metric based on symmetry considerations; a complete theory must determine $\bar{g}_{ik}$. (ii) What is the effect of quantizing the conformal part on the source—matter or radiation? (iii) Can one obtain some more information about the stationary states of geometry?

The conventional approach to quantum gravity does not make any basic distinction between the conformal degree of freedom and the background. Both the independent degrees are quantized on the same footing. By adopting such a philosophy, one can extend the concept of stationary states to all the degrees of freedom (for details of this method see Padmanabhan, 1981). As in other conventional attempts of quantum gravity, this also leads to interpretational problems.

The space-time geometry, through the light cone structure, decides the causal connection between points. Thus, any attempt to quantize all the geometrical degrees of freedom does require a reformulation of our basic concepts of causality. This problem disappears when the classical interpretation of the background metric is retained and only the conformal degree of freedom is quantized, since conformally equivalent metrics have the same light cone structure.

We explore the meaning of such a "hybrid" theory in the ensuing sections. The conformal part is quantized and the background metric is determined by a set of modified Einstein equations incorporating the expectation value of the quantized conformal past. We present some interesting results of such an attempt. This method can certainly be taken to be a better approximation than the one in which the conformal part alone is quantized.

3.2. Basic Formalism. The classical theory of gravity is obtained by varying the metric tensor g_{ik} in the action,

$$J = \frac{1}{16\pi G}\int R(-g)^{1/2}\,d^4x + \int L_m(-g)^{1/2}\,d^4x \tag{21}$$

We shall write the metric g_{ik} in the form

$$g_{ik} = \Omega^2 \bar{g}_{ik} \tag{22}$$

The action J, expressed in terms of Ω and $\bar{g}_{ik}$, has the form

$$\begin{aligned} J &= \frac{1}{16\pi G}\int \Omega^2 \bar{R}(-\bar{g})^{1/2}\, d^4x - \frac{3}{8\pi G}\int \bar{g}^{ik}\partial_i\Omega\,\partial_k\Omega(-\bar{g})^{1/2}\, d^4x \\ &\quad + \int L_m(\Omega, \bar{g})\Omega^4(-\bar{g})^{1/2}\, d^4x \\ &= \frac{1}{16\pi G}\int \Omega^2 \bar{R}(-\bar{g})^{1/2}\, d^4x + \frac{3}{4\pi G}\int \bar{L}^{\mathrm{NE}}(-\bar{g})^{1/2}\, d^4x \\ &\quad + \int L_m\Omega^4(-\bar{g})^{1/2}\, d^4x \end{aligned} \tag{23}$$

where L^{NE} has the form of the lagrangian for a *negative energy* scalar field (bar indicates quantities evaluated for the background metric). We shall now vary the conformal part Ω and the background $\bar{g}_{ik}$ as independent variables leading to the following equations:

$$\Omega^2\left(\bar{R}_{ik} - \frac{1}{2}\bar{g}_{ik}\bar{R}\right) + 6t_{ik}^{\mathrm{NE}} + \frac{16\pi G}{\Omega^2(-\bar{g})^{1/2}}\frac{\delta\left[L_m(-g)^{1/2}\right]}{\delta g^{ik}} = 0 \tag{24}$$

$$\Box\Omega + \frac{1}{6}\bar{R}\Omega = \frac{8\pi G}{3\Omega^3}\frac{1}{(-\bar{g})^{1/2}}\bar{g}^{ik}\frac{\delta\left[L_m(-g)^{1/2}\right]}{\delta g^{ik}} \tag{25}$$

Here, t_{ik}^{NE} stands for the energy-momentum tensor for a negative energy scalar field, given by

$$t_{ik}^{\mathrm{NE}} = -\partial_i\Omega\,\partial_k\Omega + \tfrac{1}{2}\bar{g}_{ik}(\partial_a\Omega\,\partial^a\Omega) \tag{26}$$

Some trivial manipulation with these equations will lead to standard Einstein equations and the constraint $\Omega = \text{const}$. This is the classical theory we expect.

We shall obtain the quantum version of the theory by the following requirements: (i) physics is described by the action in equation (23). The conformal part must be quantized, using the Hamiltonian derived from this action (this is basically what we have been doing). (ii) The background metric is not arbitrary but must be determined by equation (24) where the expressions involving Ω^2 must be replaced by the expectation values, leading to a formal equation

$$\langle\Omega^2\rangle(\bar{R}_{ik} - \tfrac{1}{2}\bar{g}_{ik}\bar{R}) + 6\langle t_{ik}^{\text{NE}}\rangle + 16\pi G\left\langle \frac{1}{\Omega^2(-\bar{g})^{1/2}} \frac{\delta\left[L_m(-g)^{1/2}\right]}{\delta g^{ik}} \right\rangle = 0 \tag{27}$$

Thus the quantization of the conformal part leads to definite quantum states. The background metric (as well as $\langle\Omega^2\rangle$) will have definite values in these states giving the space-time geometry for different quantum states. We shall show that this requirement is quite restrictive.

(Attempts have been made previously for quantizing the matter fields and using the expectation value of the stress tensor as a source for unquantized gravity. Our approach is entirely different from these. The expectation values we talk about are the expectation values in the quantum states of gravity and have nothing to do with the quantum states of matter. Moreover, we do treat part of the gravity as quantized.)

After these preliminary remarks, let us turn to the equations. The specific form of the equations depend, of course, on the form chosen for L_m. In the case of isotropic radiation, the equation for the background metric reads as

$$\langle\Omega^2\rangle(\bar{R}_{ik} - \tfrac{1}{2}\bar{g}_{ik}\bar{R}) + 6\langle t_{ik}^{\text{NE}}\rangle + 8\pi G T_{ik}^{(\text{rad})} = 0 \tag{28}$$

while the action for the (quantum) field $\Omega(x)$ has the form

$$J = -\frac{1}{16\pi G}\int(6\Omega^i\Omega_i - \bar{R}\Omega^2)\, d^4x(-\bar{g})^{1/2} \tag{29}$$

For a dust-filled universe, the corresponding equations are

$$\langle\Omega^2\rangle(\bar{R}_{ik} - \tfrac{1}{2}\bar{g}_{ik}\bar{R}) + 6\langle t_{ik}^{\text{NE}}\rangle + 8\pi G\langle\Omega\rangle T_{ik}^{(\text{dust})} = 0 \tag{30}$$

$$J = -\frac{1}{16\pi G}\int(6\Omega^i\Omega_i - \bar{R}\Omega^2)(-\bar{g})^{1/2}\, d^4x - \int\Omega T(-g)^{1/2}\, d^4x \tag{31}$$

(T stands for the trace $T_{ik}\bar{g}^{ik}$). As a last example, consider the source to be of massless scalar field. Then the equations are

$$\langle\Omega^2\rangle(\bar{R}_{ik} - \tfrac{1}{2}\bar{g}_{ik}\bar{R}) + 6\langle t_{ik}^{\text{NE}}\rangle + 8\pi G\langle\Omega^2\rangle T_{ik}^{(\text{scalar})} = 0 \tag{32}$$

$$J = -\frac{1}{16\pi G}\int(6\Omega^i\Omega_i - R\Omega^2)(-\bar{g})^{1/2}\, d^4x + \int\Omega^2 T(-\bar{g})^{1/2}\, d^4x \tag{33}$$

We shall discuss the solutions to the above equations for all the three cases, in analogy with the discussions in Section 2.

3.3. Explicit Solutions. The first surprise from the equations arises when we look for the "simplest" kind of solution—vacuum with no matter. Setting the matter part of the action to zero (i.e., $L_m = 0$) leads to the equations

$$\langle \Omega^2 \rangle \left(\bar{R}_{ik} - \tfrac{1}{2}\bar{g}_{ik}\bar{R} \right) + 6\langle t_{ik}^{\mathrm{NE}} \rangle = 0 \tag{34}$$

$$J_\Omega = -\frac{3}{8\pi G}\int \left(\Omega^i \Omega_i \right)\left(-\bar{g} \right)^{1/2} d^4x \tag{35}$$

A quantum version of equation (35) is trivial to obtain and was analyzed previously (Section 2.2). In a true vacuum, all events are equivalent and space must be homogeneous and isotropic. It is trivial to see that no such consistent solution exists for equation (34). Matterless vacuum—even flat space—is unstable against the quantum conformal fluctuations. Quantum gravity, so to say, predicts and demands the existence of matter.

Classical gravity, in this regard, is somewhat incomplete. Einstein's theory describes the evolution of matter without explaining its existence. Given a matter distribution for the universe, Einstein's equation predicts the evolution from the initial singularity without ever explaining how the matter came into being in the first place.

In fact, classical gravity cannot explain such a process of creation, since it has built into it the energy conservation law for the source. The present set of equations has an effective negative energy term $\langle t_{ik}^{\mathrm{NE}} \rangle$ which can allow for creation of matter. Quantum gravity is thus similar to the C-field theories (Hoyle and Narlikar, 1963) as far as creation of matter is concerned.

We shall now turn to the solutions in the presence of matter. We will find that the nature of the solution depends crucially on the nature of the source.

(*a*) *Radiation Universe.* Let the background metric be isotropic and homogeneous, with

$$d\bar{s}^2 = \bar{g}_{ik}\, dx^i\, dx^k = dt^2 - \frac{dr^2}{\left(1 - r^2/a^2\right)} - r^2\left(d\theta^2 + \sin^2\phi\, d^2 \right) \tag{36}$$

and the radiation source has the energy tensor

$$T_k^i = \mathrm{dia}\left(\varepsilon, -\frac{\varepsilon}{3}, -\frac{\varepsilon}{3}, -\frac{\varepsilon}{3} \right) \tag{37}$$

These symmetries restrict Ω to a function of time, whose quantization is straightforward, since the action in equation (29) corresponds to a harmonic oscillator. The stationary states are well known and characterized by the set of integers. The expectation values $\langle \Omega^2 \rangle$ and $\langle t_{ik}^{\mathrm{NE}} \rangle$ in a stationary state can be computed in a straightforward manner. A detailed algebra now reduces equation (28) to the simple, but nontrivial requirement,

$$\varepsilon = \frac{3c^4}{8\pi^2 G} \frac{L^2}{a^4} \left(n + \frac{1}{2} \right) = \frac{3}{8\pi^2} \frac{\hbar c}{a^4} \left(n + \frac{1}{2} \right) \tag{38}$$

Thus the universe can exist in quantized stationary states only if the radiation energy density is quantized. Notice that this result is obtained from the equations of gravity without any explicit quantization of the source. Classical gravity, as is well known, determines the classical dynamics of the source. Can quantum gravity uniquely determine the quantum dynamics of the source? The present result seems to suggest such an attractive possibility. However, one cannot rely too much on such a oversimplified model.

(*b*) *Dust Solution*. The same background metric can also be sustained by a dust source with the stress tensor,

$$T_{ik}^{(\mathrm{dust})} = \mathrm{dia}(\varepsilon, 0, 0, 0) \tag{39}$$

The quantization of the conformal part can again be reduced to that of a harmonic oscillator, after a shift of origin in equation (31). The stationary states are again labeled by an integer n. The energy density of dust must now satisfy the quantum condition

$$\varepsilon = \frac{c^4}{\pi^2 G} \frac{L_p}{a^3} \left(n + \frac{1}{2} \right)^{1/2} \tag{40}$$

[in analogy with equation (38)].

(*c*) *Scalar Field as Source*. The structure of stress tensor for radiation and dust [see equations (37) and (39)] is quite different from that of a negative energy scalar field which has a structure,

$$t_{ik}^{\mathrm{NE}} = \mathrm{dia}(-\rho, -\rho, -\rho, -\rho) \tag{41}$$

It is this difference in structure that leads to the self-consistency requirement in equations (38) and (40). Thus by using a scalar field as a source, one can avoid this quantum restriction and obtain a solution with *flat* background (which is not possible with radiation or dust). Assume that

$$\bar{g}_{ik} = \eta_{ik}; \qquad T_{ik}^{(\text{scalar})} = \partial_i\phi\,\partial_k\phi - \tfrac{1}{2}\bar{g}_{ik}(\phi^a\phi_a)$$
$$= \text{dia}(\varepsilon, \varepsilon, \varepsilon, \varepsilon) \tag{42}$$

It turns out that equation (32) and (33) can be consistently solved for *any* value of ε. The conformal factor behaves as a quantum harmonic oscillator with the frequency

$$\omega^2 = \left(\frac{8\pi G\varepsilon}{3}\right) \tag{43}$$

We have thus presented three different types of solutions to our equations. In the isotropic, homogeneous models there exists a scaling freedom in the form of the parameter a. Using a physical coordinate system, the metric in a stationary quantum state, labeled by n, is given by

$$ds^2 = L^2(n+\tfrac{1}{2})\left[d\eta^2 - d\chi^2 - \sin^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \tag{44}$$

which is the form originally obtained without using the full set of equations [compare with equation (15)]. Thus the lower bound to the length scale exists in all these cases and leads to the avoidance of the singularity.

4. CONCLUSION

Conformal fluctuations have proved to be of definite value in studying the structure of quantized gravity especially near the singularities. The present results about stationary states help one to understand the mechanism for the avoidance of singularity. The analogy with the hydrogen atom is now complete.

The second part of the paper has demonstrated a possible extension of the approach, at least as a valid approximation. Two major ideas emerge from this analysis: (i) Can the existence and creation of matter be explained by a quantum gravity process using the effective negative energy of the conformal part? (ii) Can the quantum dynamics of a source arise purely from a quantum gravitational consideration?

Both the questions are under investigation. [For an idea similar to (i), see Brout et al. (1978)]. Only a detailed analysis of the dynamics can answer these questions unequivocally. The present model, however, indicates a tentative possibility.

ACKNOWLEDGMENTS

This work was done under the guidance of Professor J. V. Narlikar. The author is also indebted to his colleague, Mr. K. Subramanian, for valuable discussions.

REFERENCES

Brout, R., Englert, F., and Gunzig, E. (1978). *Annals of Physics*, **115**, 78.

Feynman, R. P., and Hibbs, A. R. (1965). *Quantum Mechanics and Path Integrals*, McGraw-Hill, New York.

Hawking, S. W. (1979). In *General Relativity—An Einstein Centenary Symposium*, eds. Hawking, S. W., and Isrel, R., Cambridge U.P., Cambridge, England.

Hawking, S. W., and Ellis, G. F. R. (1973). *The Large Scale Structure of Space-Time*, Cambridge U. P., Cambridge.

Hoyle, F., and Narlikar, J. V. (1963). *Proceedings of the Royal Society, London*, **A273**, 1.

Isenberg, F., and Wheeler, J. A. (1979). In *Relativity Quanta, and Cosmology*, eds. M. Pantaleo, F. de. Finis, Johnson Reprint Corporation, New York.

Misner, C. W. (1972). In *Magic without Magic*, ed. J. R. Klander, Freeman, San Francisco.

Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1973). *Gravitation*, Freeman and Company, San Francisco.

Narlikar, J. V. (1978). *Monthly Notices of the Royal Astronomical Society*, **183**, 159.

Narlikar, J. V. (1981). *Foundations of Physics*, **11**, 473.

Padmanabhan, T. (1981). *General Relativity and Gravitation*, **13**, 451.

Padmanabhan, T. (1982). *Physics Letters*, **87A**, 226.

Padmanabhan, T., and Narlikar, J. V. (1981). *Physics Letters*, **84A**, 361.

Padmanabhan, T., and Narlikar, J. V. (1982a). *Nature*, **295**, 677.

Padmanabhan, T., and Narlikar, J. V. (1982b). Paper under preparation.

Ryan, M. (1972). "*Hamiltonian Cosmology*" (Lecture Notes in Physics 13), New York.

Schiff, S. R. (1968). *Quantum Mechanics*, McGraw-Hill, New York.

Weinberg, S. (1972). *Gravitation and Cosmology*, John Wiley, New York.

Wheeler, J. A. (1964). In *Relativity, Groups and Topology*, eds. DeWitt and DeWitt, Gordan & Breach, New York.